**Finance 6810 Assignment 1**

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Part 1

Suppose that you observe the following four bonds trading in the market.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Bond | Coupon | Time to Maturity | Price | YTM |
| A | 0 | 0.5 | 99.5 | 1.005 |
| B | 0 | 1 | 97.79 | 2.2473 |
| C | 0 | 1.5 | 96.56 | 2.3474 |
| D | 0.102 | 1.5 | 109.72 | 3.4924 |

Coupons are paid semi-annually. All four bonds have a $100 face value.

**1. Calculate zero-coupon yields for maturities of 0.5, 1-, and 1.5-years using bonds A, B, and C.**

The zero-coupon Yields to Maturity are 1.005% for bond A, 2.2473% for bond B and 2.3474% for bond C.

**2. Using the yields from (1), calculate the price of bond D if its price were consistent with bonds A, B, and C. Is bond D underpriced or overpriced?**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Term Structure of Interest Rates | | | | |
| Maturity | 0.5 | 1 | 1.5 |  |
| Rate | 1.01% | 2.25% | 2.35% |  |
|  | 5.1 | 5.1 | 105.1 |  |
| Price | 5.07 | 4.99 | 101.48 | $ 111.55 |

The price of bond D if its cashflows are discounted back to time zero using the YTM’s of zero-coupon bonds A, B & C is $111.55. At $109.72, bond D appears to be underpriced.

**3. Replicate bond D’s cash flows using a portfolio of bonds A, B, and C.**

|  |  |  |  |
| --- | --- | --- | --- |
| t1 | t2 | t3 |  |
| 0.5 | 1 | 1.5 |  |
| 5.1 | 5.1 | 105.1 |  |
| 0.051 | 0.051 | 1.051 |  |
| 5.07 | 4.99 | 101.48 | $111.55 |

I would need to buy 5.1/100 = .051 of bond A. I would need to buy 5.1/100 = .051 of Bond B, and I would need to buy 105.1/100 = 1.051 of bond C. .051 multiplied by the market price of bond A = .051 \* 99.5 = 5.07. .051 multiplied by the market price of B = .051 \* 97.79 = 4.99. 1.051 multiplied by the market price of bond C = 1.051 \* 96.56 = 101.48; 5.07 + 4.99 + 101.48 = **$111.55**

**4. Using your results in (3), construct a long-short portfolio that takes advantage of this mispricing.**

Sell short, .051 of bond A, .051 of bond B and 1.051 of bond C totaling:

(.051 \* 99.5) + (.051\* 97.79) + (1.051 \* 96.56) = 5.07 + 4.99 + 101.48 = $111.55. Use the $111.55 in short proceeds to buy bond D for $109.72. This will result in 111.55 – 109.72 = ***$1.83*** in arbitrage profits.

Part 2

Suppose that you observe the following four bonds trading in the market.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Coupon | C/2 | Time to Maturity | Price |
| A | 0.02 | 1 | 0.5 | 100.75 |
| B | 0.06 | 3 | 1 | 104.26 |
| C | 0.08 | 4 | 1.5 | 107.23 |
| D | 0.09 | 4.5 | 2 | 113.63 |

**1. Calculate zero-coupon yields for maturities of 0.5, 1, 1.5, and 2-years.**

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | Coupon | C/2 | Time to Maturity | Price | Zero Yields | Discount Factor | Pzc | YTM |
| A | 0.02 | 1 | 0.5 | 100.75 | 0.496% | 0.997524752 | 100.75 | 0.496% |
| B | 0.06 | 3 | 1 | 104.26 | 1.704% | 0.983178891 | 101.27 | 1.69% |
| C | 0.08 | 4 | 1.5 | 107.23 | 3.102% | 0.954876783 | 99.31 | 3.03% |
| D | 0.09 | 4.5 | 2 | 113.63 | 2.001% | 0.960955867 | 100.42 | 2.01% |

**2. Calculate the discount factors that the zero-coupon yields imply. Do you see any potential problems? Why?**

|  |
| --- |
| Discount Factor |
| 0.997524752 |
| 0.983178891 |
| 0.954876783 |
| 0.960955867 |

The problem I see with the discount factors is that bond ‘C’ has a lower discount factor than bond ‘D’ despite bond ‘D’ having a longer time to maturity. Similarly, the zero-yield curve is distorted regarding the 1.5 and 2-year premium bonds—the 1.5-year bond offers a higher zero yield (***3.102%****)*than the 2-year bond despite the 2-year bond having a longer maturity. This is a violation of the term premium convention.

**3. Suppose that you have a technology that allows you to store money for free (a “mattress”) between years 1.5 and 2. That is, if you put $x under your mattress at t = 1.5, you will still have $x at t = 2. Construct a long-short trading strategy using the four bonds that earns you free money today.**

**Hint: Start by replicating bond D with a portfolio of bonds A, B, and C, along with the Mattress Technology. In particular, 1 unit of the “mattress” technology has the following cash flows:**

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Description automatically generated with medium confidence**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | **Units** | **T = 0** | **T = .5** | **T = 1** | **T = 1.5** | **T = 2** |
| **Mattress Tech.** | **1** | **0** | **0** | **0** | **-1** | **+1** |

I was able to replicate bond D for a price of $113.00. Bond D’s market price is $113.63. Thus, to make arbitrage profits today, I would need to sell bond D short for $113.63 and use those proceeds to buy the replicated bond D. This would result in 113.63 – 113 = ***$.63*** in arbitrage profits.

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Similarly, I was able to use the zero yields from problem 1 to discount bond D’s cashflows to time zero. The result is 4.49 + 4.42 + 104.08 = **$113** which is the same as the cost of my replicated bond D.